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Also for $x$ negative Eq. (4) assumes the arithmetical form

$$\frac{p^n}{q^n} = \frac{x^n}{1-x}. \tag{10}$$

The value of $x$ in Eq. (9) can lie any where between 0 and infinity; but the value of $x$ in Eq. (10) can only lie between 0 and 1. This proves IV. And Prop. V. also follows from the foregoing.

*Note.* The cases in which $n$ is fractional, and, in which $p$ and $q$ (both or either) are imaginary, have not, though they might have, been considered.

Also, an equation of the form, $z^n + p'z^{n-1} + q' = 0$ is merely the reciprocal form of Eq. (1). For if $z = \frac{1}{y}$, then $y^n + \frac{p'}{q'}y + \frac{1}{q'} = 0$, which is the same as Eq. (1).

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## *THE CENTRE OF GRAVITY OF THE APPARENT DISK OF A PLANET.*

BY PROF. A. HALL, U. S. NAVAL OBSERVATORY, WASHINGTON, D. C.

When a planet is observed from the Earth its disk will not be completely illuminated except at the time of opposition. At other times it will appear gibbous or crescent, If we assume the planet to be a sphere, which, according to the best observations is the case with Mercury, Venus and Mars, the apparent disk will be composed of a semi-circle and a semi-ellipse which are separated by the line of cusps. If a right line be drawn through the centre of the line of cusps and at right angles to it, the centre of gravity of the apparent disk is in this line. In the plane triangle between the Sun, the Earth and the planet let $\varphi$ be the angle at the planet; let $a$ be the radius of the semi-circular part of the disk, and designate by $m$ the distance of the centre of gravity from the centre of the line of cusps. Then when the planet is gibbous we shall have

$$m = \frac{8a}{3\pi}\sin\tfrac{1}{2}\varphi^2;$$

and when the planet is crescent,

$$m = \frac{8a}{3\pi}\cos\tfrac{1}{2}\varphi^2,$$

where $\pi$ is the number 3.1416. The angle $\varphi$ and the radius $a$ can be computed from the data given in the ephemerides, and the distance $m$ is known therefore for any time.

In order to show the use of the quantity $m$, suppose the angle of position and distance of a satellite of a planet have been observed with a filar micrometer in the following manner. The wire is laid across the apparent disk of the planet in such a way that this disk is divided into two parts of equal area. The wire therefore always passes through the centre of gravity of the apparent disk, and it is necessary before using the observations for computing orbits to reduce these measures to the true centre of the planet. If we denote by $p$ and $s$ the observed angle of position and the distance of the satellite, and by $\theta$ the angle of position of the line of cusps, the errors of $p$ and $s$ arising from the unsymmetrical form of the disk will be,

$$\Delta s = m \, . \sin(p-\theta),$$

$$\Delta p = \frac{m}{s} . \cos(p-\theta),$$

where $\Delta p$ is expressed in parts of radius.

To complete the solution we have to find the value of $\theta$. This is given from the spherical triangle between the pole of the Equator and the geocentric places of the Sun and the planet. In this triangle the angle at the planet is $90°-\theta$; and if we denote by $d$ the angular distance between the Sun and the planet, and by $a$, $\delta$, $a'$, $\delta'$, the geocentric right ascensions and declinations of the Sun and planet, we shall have,

$$\begin{aligned} \cos d &= \sin\delta \sin\delta' + \cos\delta \cos\delta' \cos(a'-a), \\ \sin d \cos\theta &= \cos\delta \sin(a'-a), \\ \sin d \sin\theta &= \sin\delta \cos\delta' - \cos\delta \sin\delta' \cos(a'-a). \end{aligned}$$

These equations give the angle $\theta$ without ambiguity.

In the case of a spheroidal planet, as Jupiter or Saturn, the apparent outline of the planet will be composed of ellipses, but the determination of these ellipses though possible is much more troublesome, and for these planets the correction for defective illumiuation is hardly sensible.

## *A PROBLEM AND ITS SOLUTION.*

BY E. B. SEITZ, GREENVILLE, OHIO.

On the surface of a sphere, radius unity, three small circles, radii $a$, $b$, $c$, are described, each touching the other two; on the surface enclosed by them three small circles, radii $x_1$, $y_1$, $z_1$, are described, each touching the other two and two of the first set of circles; on the surface enclosed by the first set of inscribed circles three small circles, radii $x_2$, $y_2$, $z_2$, are described,